Verification of Current Source

An ideal current source has been incorporated into the XOOPIC main tree, with the purpose of sustaining a DC discharge. In order to verify the current source, the computational implementation is compared with analytic model.

1 Current Source Formulation: Cartesian Coordinates

Consider an ideal current source at the left hand wall. Let there be N nodes on this wall. The potential on the wall, Φ_0, is a function of the current applied and the plasma parameters.

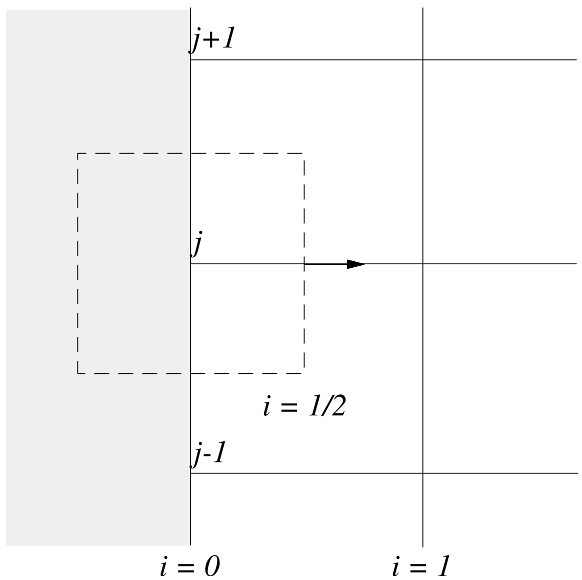

Figure 1: Gaussian volume

Choosing a Gaussian volume equal to a cell size centered at node $i = 0, j$ on the (ideal) conductor, the charge enclosed is equal to the flux out of the volume. This volume is shown in Figure 1. Since there is no field tangential to the conductor's surface or internal to the conductor,

$$dyE_x = \frac{dy}{dx}(\Phi_0 - \Phi_{1,j}) = \frac{Q_j}{\epsilon} \tag{1}$$

i and j are counters for the x and y directions, respectively. Q_j is the total charge enclosed in the volume. dx and dy are the mesh spacings in the x and y directions, respectively. in the y direction.

$$Q_j = Q_{(wall)j} + Q_{(plasma)j} \tag{2}$$

$Q_{(plasma)j}$ is the charge due to the plasma in the region, and is known in PIC. $Q_{(wall)j}$ is the charge on the wall in the volume associated with indices (i, j) and is unknown, but subject to the condition

$$\sum_{j}^{N} Q_{(wall)j} = Q_T \tag{3}$$

Q_T is the total charge on the wall. This is known from current continuity and includes charge convected to the wall.

Using the principle of superposition, the potential $\Phi_{i,j}$ may be written as

$$\Phi_{i,j} = \Phi_{(P)i,j} + \Phi_0 \Phi_{(L)i,j} \tag{4}$$

Φ_P is the potential due to the plasma with the conductor grounded, and $\Phi_{(L)}$ is the potential from a solution of Laplace's equation with the conductor biased to a unit potential. Both Φ_P and Φ_L are known.

Summing over index j, one obtains

$$\frac{dy}{dx} \sum_{j}^{N} \Phi_0 - \Phi_{1,j} = \frac{1}{\epsilon} \sum_{j}^{N} Q_j \tag{5}$$

$$\Phi_0 = \frac{1}{N} \left(\sum_{j}^{N} \left[\Phi_{(P)1,j} + \Phi_0 \Phi_{(L)1,j} \right] + \frac{dx}{\epsilon dy} \left[Q_T + \sum_{j}^{N} Q_{(plasma)j} \right] \right) \tag{6}$$

$$\Phi_0 - \frac{\Phi_0}{N} \sum_{j}^{N} \Phi_{(L)1,j} = \frac{1}{N} \left(\sum_{j}^{N} \Phi_{(P)1,j} + \frac{dx Q_T}{\epsilon dy} + \frac{dx}{\epsilon dy} \sum_{j}^{N} Q_{(plasma)j} \right) \tag{7}$$

$$\Phi_0 = \frac{\sum_{j}^{N} \Phi_{(P)1,j} + \frac{dx Q_T}{\epsilon dy} + \frac{dx}{\epsilon dy} \sum_{j}^{N} Q_{(plasma)j}}{N - \sum_{j}^{N} \Phi_{(L)1,j}} \tag{8}$$

2 Current Source Formulation: Cylindrical Coordinates and Generalized Treatment

Due to geometric factors, the above treatment does not suffice for cylindrical coordinates. It is convenient to derive equations that work in both cylindrical and Cartesian coordinates. With similar conventions, the above equations become

$$A_j E_x = \frac{A_j}{dx} (\Phi_0 - \Phi_{1,j}) = \frac{Q_j}{\epsilon} \tag{9}$$

$$Q_j = Q_{(wall)j} + Q_{(plasma)j} = Q_{(wall)j} + V_j \rho_{(plasma)j} \tag{10}$$

$$\sum_{j}^{N} Q_{(wall)j} = Q_T \tag{11}$$

$$\Phi_{i,j} = \Phi_{(P)i,j} + \Phi_0 \Phi_{(L)i,j} \tag{12}$$

Summing over index j, one obtains

$$\frac{1}{dx} \sum_j^N A_j (\Phi_0 - \Phi_{1,j}) = \frac{1}{\epsilon} \sum_j^N Q_j \tag{13}$$

$$\sum_j^N A_j \left(\Phi_0 - \Phi_0 \Phi_{(L)1,j} - \Phi_{(P)1,j} \right) = \frac{dx}{\epsilon} \left(Q_T + \sum_j^N V_j \rho_{(plasma)j} \right) \tag{14}$$

$$\Phi_0 = \frac{\sum_j^N A_j \Phi_{(P)1,j} + \frac{dx}{\epsilon} \left(Q_T + \sum_j^N V_j \rho_{(plasma)j} \right)}{\sum_j^N A_j \left(1 - \Phi_{(L)j} \right)} \tag{15}$$

Note that since A is left as a general expression for area, this treatment is equally applicable to Cartesian coordinates. Also note that this treatment is valid for the left hand wall and the lower wall (or inner wall, in cylindrical coordinates). Since the normal vector has the opposite sign convention for the right hand wall and upper (outer) wall, the sign on the charge terms must be reversed for these cases. The treatment is then generally valid.

3 Corner Effects

The above treatment has assumed a Gaussian pillbox for a point internal to a conductor. Usually, a conductor will have two "corner" points that do not follow the treatment given in Equations 1 and 9, as the normal vectors at these corners are undefined.

To implement this in XOOPIC, we assume that the starting and ending cells given for the current source are filled half with the driven conductor and half with a dielectric of unity relative dielectric constant. This configuration is shown for the "top" corner in Figure 2.

The above solution will be amended for a conductor with nodes running from $Jmin$ to $Jmax$. The additional flux out in the y direction is from the nodes $Jmin$ and $Jmin + 1$ as well as from $Jmax$ and $Jmax - 1$:

$$\begin{aligned} A_y E_y = & \frac{A_{y,Jmax}}{dy} \left(\Phi_0 \left[1 - \Phi_{(L)0,Jmax} \right] - \Phi_{(P)0,Jmax} \right) \\ & - \frac{A_{y,Jmin}}{dy} \left(\Phi_0 \left[1 - \Phi_{(L)0,Jmin} \right] - \Phi_{(P)0,Jmin} \right) \end{aligned} \tag{16}$$

Adding Equation 16 with Equation 14 and adding definite limits to the sums, one obtains an expression for Φ_0:

$$\sum_{j=Jmin+1}^{Jmax-1} \frac{A_j}{dx} \left(\Phi_0 - \Phi_0 \Phi_{(L)1,j} - \Phi_{(P)1,j} \right)$$

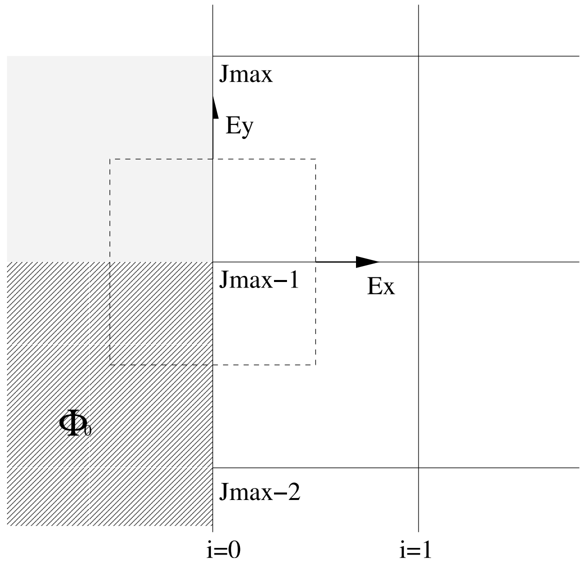

Figure 2: Gaussian volume for the top left corner

$$+\frac{A_{y,Jmax}}{dy}\left(\Phi_0\left[1-\Phi_{(L)0,Jmax}\right]-\Phi_{(P)0,Jmax}\right)$$
$$-\frac{A_{y,Jmin}}{dy}\left(\Phi_0\left[1-\Phi_{(L)0,Jmin}\right]-\Phi_{(P)0,Jmin}\right)$$
$$=\frac{1}{\epsilon}\left(Q_T+\sum_{j=Jmin+1}^{Jmax-1}V_j\rho_{(plasma)j}\right) \tag{17}$$

Solving for Φ_0

$$\Phi_0=\frac{\frac{A_{y,Jmax}\Phi_{(P)0,Jmax}}{dy}-\frac{A_{y,Jmin}\Phi_{(P)0,Jmin}}{dy}+\sum_{j=Jmin+1}^{Jmax-1}\frac{A_j}{dx}\Phi_{(P)1,j}+\frac{1}{\epsilon}\left(Q_T+\sum_{j=Jmin+1}^{Jmax-1}V_j\rho_{(plasma)j}\right)}{\frac{A_{y,Jmax}(1-\Phi_{(L)0,Jmax})}{dy}-\frac{A_{y,Jmin}(1-\Phi_{(L)0,Jmin})}{dy}+\sum_{j=Jmin+1}^{Jmax-1}\frac{A_j}{dx}\left(1-\Phi_{(L)j}\right)} \tag{18}$$

4 Parallel Plate Capacitor

Kirchoff's laws gives the expression for voltage on a capacitor as a function of DC current:

$$V=\frac{Q}{C}=\frac{1}{C}\int_0^t Idt'=\frac{It}{C} \tag{19}$$

This assumes the initial charge on the capacitor is zero. For a parallel plate capacitor with $\epsilon = \epsilon_0$

$$C = \frac{A\epsilon}{d} => V = \frac{Idt}{\epsilon_0 A} \tag{20}$$

For $A = 7.85398E - 5m^2$, $I = 0.001A$, $\epsilon = 8.8542E - 12F/m$, and $d = 0.01$, the voltage is

$$V = 1.4380E10t \tag{21}$$

The voltage calculated using this geometry configuration and others compared well with theory.

5 Initial Voltage

Often, it is desirable to start an electrode with some initial potential. With the preceeding current source formulation, it is necessary to find the equivalent total charge at $t = 0$ for the specified voltage V_i. This is accomplished through manipulation of Equation 18: